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Pairwise G_F −Completely Continuous Maps

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Abstract: In this paper, we have studied the concept of pairwise $\mathbf{G_F - continuous\ map}$, somewhat pair-wise* fuzzy continuous maps, pairwise somewhat $\mathbf{G_F}$ −completely continuous maps in $\mathbf{G_F - topological\ spaces}$ and verify the results with counter examples. Further we have established the relationships of these maps and characterize them suitably.

Keywords: $G_F -$ bi-topological space, somewhat pair-wise $G_F -$ completely continuous map, pair-wise* G_F −completely continuous map

1. Introduction

Zadeh (1965) has introduced the notion of fuzzy set which is a significant notion in the theory of fuzzy mathematics. Chang (1968) has introduced the concept of fuzzy topological space as a generalization of topological space and Kandil (1989) introduced fuzzy bi-topological spaces in 1989. Gentry et.al. (1971) have defined the concept of somewhat continuous maps in topological space. In this sequence Thangraj (2003) introduced the concept of somewhat fuzzy continuous maps. Bhaumik and Mukherjee (1993) have defined the concept of fuzzy completely continuous maps, Swaminathanand Vadivel (2017-18) have defined some-what fuzzy completely continuous maps, somewhat pair-wise fuzzy continuous maps, somewhat pair-wise fuzzy α-irresolute map and somewhat pair-wise fuzzy irresolute map.

Organization: The rest of the paper structured as follows: Some require basic definitions, concepts of G_F −bi-topological space and notations are discussed in Section 2. The section 3 has been headed by "pairwise G_F −continuous maps" in which we discuss the concept of continuity and established the several relationships by making the use of some examples. In section 4 headed by "somewhat pair-wise* fuzzy continuous maps" we studied several results with the help of supportive examples. In section 5 headed by "pairwise somewhat G_F −completely continuous map" we discusses and characterise the several maps and establish the relationships with the help of some counter examples. Finally, Section 6 concludes the paper.

2. Preliminaries

Definition 2.1: Let (X, T_1, T_2) consisting of a universal set X with the G_F −topologies T_1 and T_2 on X is called G_F −bi-topological space. A fuzzy set 'P' of G_F −bi-topological space (X, T_1, T_2) is called $G_F - (i,j) - \text{semi} - \text{open set}$ if $P \subseteq T_j - Cl_F(T_i - I_F(P))$. A fuzzy set 'P' is called $G_F - (i,j) - \text{semi} - \text{closed set}$ if $P^c = 1 - P$ is $G_F - (i,j) - \text{semi} - \text{open set}$

Definition 2.2: A fuzzy set 'P' of G_F −bi-topological space (X, T_1, T_2) is called $G_F - (i,j) - \text{pre} - \text{open set}$ if $P \subseteq T_i - I_F(T_j - Cl_F(P))$. A fuzzy set 'P' is called $G_F - (i,j) - \text{pre} - \text{closed set}$ if $P^c = 1 - P$ is $G_F - (i,j) - \text{pre} - \text{open set}$.

Definition 2.3: A fuzzy set 'P' of G_F −bi-topological space (X, T_1, T_2) is called $G_F - (i,j) - \alpha - \text{open set}$ if $P \subseteq T_i - I_F(T_j - Cl_F(T_i - I_F(P)))$. A fuzzy set 'P' is called $G_F - (i,j) - \alpha - \text{closed set}$ if $P^c = 1 - P$ is $G_F - (i,j) - \alpha - \text{open set}$

Definition 2.4: A fuzzy set 'P' of G_F −bi-topological space (X, T_1, T_2) is called $G_F - (i,j) - \beta - \text{open set}$ if $P \subseteq T_j - Cl_F(T_i - I_F(T_j - Cl_F(P)))$. A fuzzy set 'P' is called $G_F - (i,j) - \beta - \text{closed set}$ if $P^c = 1 - P$ is $G_F - (i,j) - \beta - \text{open set}$.

Definition 2.5: A fuzzy set 'P' of G_F −bi-topological space (X, T_1, T_2) is called
i) $G_F - (i,j) - \text{regular} - \text{open set}$ if $T_j - I_F(T_i - Cl_F(P)) = P$
ii) $G_F - (i,j) - \text{regular} - \text{closed set}$ if $T_j - Cl_F(T_i - I_F(P)) = P$

Remark 2.1: In a G_F −bi-topological space (X, T_1, T_2)
i) Every $G_F - (i,j) - \text{regular} - \text{open set}$ is $G_F - T_j - \text{open}$ but not converse

ii) Every $G_F-(i,j)-$ regular $-$ closed set is G_F-T_j- closed but not converse

3. Pairwise G_F − continuous map

Definition 3.1: A mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is said to be pairwise G_F −continuous map if $F:(X,T_1)\to(Y,J_1)$ and $F:(X,T_2)\to(Y,J_2)$ are G_F −continuous map

Example 3.1: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$ .Consider fuzzy sets $A_1=\{(x_1/0.3),(x_2/0.6)\}$, $A_2=\{(x_1/0.5),(x_2/0.4)\}$, $A_3=\{(x_1/0.5),(x_2/0.6)\}$, $B_1=\{(x_1/0.2),(x_2/0.5)\}$, $B_2=\{(x_1/0.6),(x_2/0.3)\}$ and $B_3=\{(x_1/0.6),(x_2/0.5)\}$ on X. Again $C_1=\{(y_1/0.3),(y_2/0.6)\}$, $C_2=\{(y_1/0.5),(y_2/0.4)\}$, $C_3=\{(y_1/0.5),(y_2/0.6)\}$, $D_1=\{(y_1/0.2),(y_2/0.5)\}$, $D_2=\{(y_1/0.6),(y_2/0.3)\}$ and $D_3=\{(y_1/0.6),(y_2/0.5)\}$ on Y. Let $T_1=\{0,A_1,A_2,A_3,1\}$, $T_2=\{0,B_1,B_2,B_3,1\}$, $J_1=\{0,C_1,C_2,C_3,1\}$ and $J_2=\{0,D_1,D_2,D_3,1\}$ be the G_F −topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ such that $F(x_1)=y_1$ and $F(x_2)=y_2$ which is pairwise G_F −continuous map.

Definition 3.2: A mapping $F:(X,T_1,T_2)\to(Y,J_3,J_4)$ is said to be pairwise G_F −semi-continuous map if the inverse image of J_i-G_F −open sets in Y is $G_F-(i,j)$ −semi-open set in X. In Example 3.1, the mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ defined as $F(x_1)=y_1$ and $F(x_2)=y_2$ is pairwise G_F −semi-continuous map

Remark 3.1: In a G_F- bi-topological space (X,T_1,T_2) every pairwise G_F −continuous map **is** G_F −semi-continuous map but not converse in general which is shown in Example 3.2

Example 3.2: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$ .Consider fuzzy sets $A_1=\{(x_1/0.3),(x_2/0.6)\}$, $A_2=\{(x_1/0.5),(x_2/0.4)\}$, $A_3=\{(x_1/0.5),(x_2/0.6)\}$, $B_1=\{(x_1/0.2),(x_2/0.5)\}$, $B_2=\{(x_1/0.6),(x_2/0.3)\}$ and $B_3=\{(x_1/0.6),(x_2/0.5)\}$ on X. Again $C_1=\{(y_1/0.3),(y_2/0.6)\}$, $C_2=\{(y_1/0.5),(y_2/0.4)\}$, $C_3=\{(y_1/0.5),(y_2/0.6)\}$, $C_4=\{(y_1/0.6),(y_2/0.6)\}$, $D_1=\{(y_1/0.2),(y_2/0.5)\}$, $D_2=\{(y_1/0.6),(y_2/0.3)\}$ and $D_3=\{(y_1/0.6),(y_2/0.5)\}$ on Y. Let $T_1=\{0,A_1,A_2,A_3,1\}$, $T_2=\{0,B_1,B_2,B_3,1\}$, $J_1=\{0,C_1,C_2,C_3,C_4,1\}$ and $J_2=\{0,D_1,D_2,D_3,1\}$ be the G_F −topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ such that $F(x_1)=y_1$ and $F(x_2)=y_2$ which is pairwise G_F −semi-continuous map but not pairwise G_F −continuous map because the set $C_4=\{(y_1/0.6),(y_2/0.6)\}$ is J_1-G_F- open set in Y but not T_i-G_F- open set $(i=1,2)$ in X.

Definition 3.3: A mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is said to be pairwise G_F- pre $-$ continuous map if inverse image of every J_i-G_F- open set in Y is $G_F-(i,j)-$ pre $-$ open set in X

Example 3.4: In Example 3.1, the mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ defined as $F(x_1)=y_1$ and $F(x_2)=y_2$ is pairwise G_F- pre $-$ continuous map

Remark 3.2: In a G_F- bi-topological space (X,T_1,T_2) every pairwise G_F −continuous map **is** G_F −pre-continuous map but not converse in general which is shown in Example 3.5

Example 3.5: In Example 3.2, consider$C_4=\{(y_1/0.5),(y_2/0.3)\}$ and $T_1=\{0,A_1,A_2,A_3,1\}$, $T_2=\{0,B_1,B_2,B_3,1\}$, $J_1=\{0,C_1,C_2,C_3,C_4,1\}$ and $J_2=\{0,D_1,D_2,D_3,1\}$ be the G_F-topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ such that $F(x_1)=y_1$ and $F(x_2)=y_2$ which is pairwise G_F −pre-continuous map but not pairwise G_F −continuous map because the set $C_4=\{(y_1/0.5),(y_2/0.3)\}$ is J_1-G_F- open set in Y but not T_i-G_F- open set $(i=1,2)$ in X.

4. Somewhat Pair-wise* Fuzzy Continuous Maps

Definition 4.1: A mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is said to be somewhat pairwise fuzzy continuous map if there exists T_1- fuzzy open set or T_2- fuzzy open set $A\neq 0_x$ on (X,T_1,T_2) such that $A\subseteq F^{-1}(B)\neq 0_x$ for any J_1- fuzzy open set or J_2- fuzzy open set $B\neq 0_y$

Example 4.1: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$. Suppose and A, B, C and D are fuzzy sets on X and Y which are defined as $A=\{(x_1/0.4),(x_2/0.5)\}$, $B=\{(x_1/0.3),(x_2/0.4)\}$, $C=\{(y_1/0.4),(y_2/0.4)\}$and $D=\{(x_1/0.5),(x_2/0.5)\}$ Let $T_1=\{0,A,1\}$, $T_2=\{0,B,1\}$, $J_1=\{0,C,1\}$ and $J_2=\{0,D,1\}$ be two fuzzy topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\to(X,J_1,J_2)$ such that $F(x_1)=y_1$ and $F(x_2)=y_2$. We observe that T_1- fuzzy open set $B\neq 0_x$, such that $B\subseteq F^{-1}(C)$ and $B\subseteq F^{-1}(D)$. Thus the mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is somewhat pairwise fuzzy continuous map.

Definition 4.2: A fuzzy set A on a fuzzy bi-topological space (X,T_1,T_2) is called pairwise dense fuzzy set if there exists no T_1- fuzzy closed set or T_2- fuzzy closed set B such that $A\subseteq B\subseteq 1$

Definition 4.3: A mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is said to be somewhat pairwise fuzzy pre-irresolute map if there exists T_1- fuzzy open set or T_2- fuzzy open set $A\neq 0_x$ on (X,T_1,T_2) such that $A\subseteq F^{-1}(B)\neq 0_x$ for any J_1- fuzzy pre $-$ open set or J_2- fuzzy pre $-$ open set $B\neq 0_y$ on (Y,J_1,J_2)

Example 4.2: Let $X=\{x_1,x_2,x_3\}$ and $Y=\{y_1,y_2,y_3\}$. Suppose and A, B, C and D are fuzzy sets on X and Y which are defined as $A=\{(x_1/0.1),(x_2/0.1),(x_3/0.1)\}$, $B=\{(x_1/0.3),(x_2/0.3),(x_3/0.3)\}$, $C=\{(x_1/0.8),(x_2/0.8),(x_3/0.8)\}$and $D=\{(x_1/0.9),(x_2/0.9),(x_3/0.9)\}$ Let $T_1=\{0,A,1\}$, $\mathcal{T}_2=\{0,B,1\}$, $J_1=\{0,C,1\}$ and $J_2=\{0,D,1\}$ be two fuzzy topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\to(X,J_1,J_2)$ such that $F(x_1)=y_1$, $F(x_2)=y_2$ and $F(x_3)=y_3$. Then we have $A\subseteq F^{-1}(C)$ and $A\subseteq F^{-1}(D)$. Since A is T_1- fuzzy pre $-$ open set on (X,T_1,T_2), therefore the mapping $F:(X,T_1,T_2)\to(Y,J_1,J_2)$ is somewhat pairwise fuzzy pre-irresolute map

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Definition 4.4: A mapping $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is said to be pairwise fuzzy almost continuous map if $F^{-1}(A)$ is T_i- fuzzy open set in X for every fuzzy $(i,j)-$ regular $-$ open set in Y

Example 4.3: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$. Suppose and A, B, C and D are fuzzy sets on X and Y which are defined as $A=\{(x_1/0.6),(x_2/0.5)\}$, $B=\{(x_1/0.3),(x_2/0.5)\}$, $C=\{(x_1/0.5),(x_2/0.4)\}$, $D=\{(y_1/0.6),(y_2/0.5)\}$ and $E=\{(y_1/0.3),(y_2/0.5)\}$ Let $T_1=\{0,A,C,1\}$, $T_2=\{0,B,1\}$, $J_1=\{0,C,1\}$ and $J_2=\{0,D,E,1\}$ be two fuzzy topologies defined on X and Y. Then we define a mapping $F:(X,T_1,T_2)\rightarrow(X,J_1,J_2)$ such that $F(x_1)=y_1$ and $F(x_2)=y_2$. Then we see that 0, 1 and E are fuzzy $(i,j)-$ regular $-$ open set in Y, also $F^{-1}(0)$, $F^{-1}(1)$ and $F^{-1}(E)=B$ are T_2- fuzzy open set in X. Thus the mapping $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is pairwise fuzzy $(1,2)-$ almost continuous map

Definition 4.5: A mapping $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is said to be pair-wise* fuzzy continuous map if $F^{-1}(A)$ is a T_1- fuzzy open set or T_2- fuzzy open set on (X,T_1,T_2) for any J_1- fuzzy open set or J_2- fuzzy open set A on (Y,J_1,J_2)

Proposition 4.1: Every pair-wise fuzzy continuous map is pair-wise* fuzzy continuous map.

Proof: Suppose $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is pair-wise fuzzy continuous map. Then by definition for each $A\in J_1$, $F^{-1}(A)\in T_1$. This implies that $F^{-1}(A)$ is a T_1- fuzzy open set or T_2- fuzzy open set on (X,T_1,T_2) for any any J_1- fuzzy open set or J_2- fuzzy open set A on (Y,J_1,J_2). Thus $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is pair-wise* fuzzy continuous map.

Remark 4.1: Converse of the Proposition 4.1 is not true in general which can be shown in Example 4.4

Example 4.4`: Let A, B, C be fuzzy sets on $X=\{x_1,x_2,x_3\}$ and D, E be fuzzy sets on $Y=\{y_1,y_2,y_3\}$, which are defined as $A=\{(x_1/0.4),(x_2/0.5),(x_3/0.5)\}$ $B=\{(x_1/0.5),(x_2/0.5),(x_3/0.5)\}$, $C=\{(x_1/0.5),(x_2/0.6),(x_3/0.6)\}$, $D=\{(y_1/0.4),(y_2/0.5),(y_3/0.4)\}$ and $E=\{(y_1/0.5),(y_2/0.6),(y_3/0.6)\}$ Let $T_1=\{0,A,B,1\}$, $T_2=\{0,C,1\}$, $J_1=\{0,D,1\}$ and $J_2=\{0,E,1\}$ be two fuzzy topologies defined on X and Y. Thus we can see that for $F^{-1}(D)\in T_1$ and $F^{-1}(E)\in T_1$ and hence $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is pair-wise* fuzzy continuous map, but $F^{-1}(E)\notin T_2$, therefore $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is not pair-wise fuzzy continuous map.

Definition 4.6: A mapping $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$is said to be pair-wise* fuzzy completely continuous map if $F^{-1}(A)$ is T_1-fuzzy regular open or T_2-fuzzy regular open set on (X, T_1, T_2) for any J_1-fuzzy open or J_2-fuzzy open set A in Y.

Proposition 4.2: Every pair-wise* fuzzy completely continuous map is pair-wise* fuzzy continuous map.

Proof: Let $(X,T_1,T_2)$and (Y,J_1,J_2) be fuzzy bi-topological spaces and $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ be a pair-wise* fuzzy completely continuous map. Suppose A is a J_1-fuzzy open set or J_2-fuzzy open set in Y. Since "F" is a pair-wise* fuzzy completely continuous map $F^{-1}(A)$is T_1-fuzzy regular open set or T_2-fuzzy regular open set in Y. Since each regular open set is fuzzy open it follows that f is pair-wise* fuzzy continuous map.

Remark 4.2: The converse of the above Proposition 4.2 may not necessarily true as shown in Example 4.5.

Example 4.5: Let A, B, C, D be fuzzy sets on $X=\{x_1,x_2,x_3\}$ and E, F, G be fuzzy sets on $Y=\{y_1,y_2,y_3\}$, which are defined as $A=\{(x_1/0.5),(x_2/0.5),(x_3/0.5)\}$ $B=\{(x_1/0.4),(x_2/0.4),(x_3/0.5)\}$, $C=\{(x_1/0.3),(x_2/0.3),(x_3/0.3)\}$, $D=\{(x_1/0.4),(x_2/0.4),(x_3/0.4)\}$, $E=\{(y_1/0.4),(y_2/0.4),(y_3/0.4)\}$ $F=\{(y_1/0.4),(y_2/0.4),(y_3/0.5)\}$ and $G=\{(y_1/0.3),(y_2/0.3),(y_3/0.3)\}$ Let $T_1=\{0,A,B,1\}$, $T_2=\{0,C,D,1\}$, $J_1=\{0,E,F,1\}$ and $J_2=\{0,G,1\}$ be two fuzzy topologies defined on X and Y. Now we define a map $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ such that $F(x_1)=y_1$, $F(x_2)=y_2$, $F(x_3)=y_3$. Here we can see that $F^{-1}(G)$ is T_2-fuzzy open, $F^{-1}(F)$ is T_1-fuzzy open and $F^{-1}(E)$ is T_1-fuzzy open, hence "F" is pair-wise* fuzzy continuous map. But $F^{-1}(F)$ is not fuzzy regular open, hence "F" is not pair-wise* fuzzy completely continuous map.

Definition 4.7: A mapping $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is called somewhat pair-wise* fuzzy completely continuous map if there exist T_1-fuzzy regular open or T_2-fuzzy regular open set $A\neq 0_X$ on (X,T_1,T_2) such that $A\subseteq F^{-1}(B)\neq 0_X$ for any J_1-fuzzy open or J_2-fuzzy open set $B\neq 0_Y$ in Y.

Proposition 4.3: Every pair-wise* fuzzy completely continuous map is somewhat pair-wise fuzzy completely continuous map

Proof: Let $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ be a pair-wise* fuzzy completely continuous map. Suppose $B\neq 0_Y$ is a J_1-fuzzy open set or J_2-fuzzy open set in Y and $F^{-1}(B)\neq 0_X$. Since $F:(X,T_1,T_2)\rightarrow(Y,J_1,J_2)$ is pair-wise* fuzzy completely continuous map, $F^{-1}(B)$ is a T_1-fuzzy regular open set or T_2-fuzzy regular open set in X.

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Taking $A = F^{-1}(B)$, it follows that $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is a somewhat pair-wise fuzzy completely continuous map.

Remark 4.3: The converse of above Proposition 4.3 may not necessarily true as shown in Example 4.6.
Example 4.6: Let A_1, A_2 and A_3 be fuzzy sets on $X = \{x_1, x_2, x_3\}$ and B_1, B_2 and B_3 be fuzzy sets on $Y = \{y_1, y_2, y_3\}$, which are defined as $A_1 = \{(x_1/0.4), (x_2/0.4), (x_3/0.4)\}$, $A_2 = \{(x_1/0.1), (x_2/0.1), (x_3/0.1)\}$, $A_3 = \{(x_1/0.5), (x_2/0.5), (x_3/0.5)\}$, $B_1 = \{(y_1/0.5), (y_2/0.5), (y_3/0.5)\}$, $B_2 = \{(y_1/0.3), (y_2/0.1), (y_3/0.3)\}$, $B_3 = \{(y_1/0.5), (y_2/0.4), (y_3/0.5)\}$. Consider fuzzy topologies $T_1 = \{0, {A_2}^c, 1\}$, $T_2 = \{0, A_2, 1\}$, $J_1 = \{0, B_1, B_2, 1\}$ and $J_2 = \{0, B_3, 1\}$. Now we define a map $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ such that $F(x_1) = y_2$, $F(x_2) = y_2$, $F(x_3) = y_2$. Then we have $A_2 \subseteq F^{-1}(B_1) = A_3 A_2 \subseteq F^{-1}(B_2) = A_2 A_2 \subseteq F^{-1}(B_3) = A_1$. Since A_2 is T_1 −fuzzy regular open, hence $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is somewhat pair-wise fuzzy completely continuous map. But $F^{-1}(B_1)$ and $F^{-1}(B_3)$ are not fuzzy regular open so $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is not pair-wise* fuzzy completely continuous map.

5. Pairwise Somewhat G_F −Completely Continuous Map
Definition 5.1: Let (X, T_1, T_2) and (Y, J_1, J_2) be G_F −bi-topological space. A mapping $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is said to be pairwise G_F −completely continuous map if inverse image of every $J_i - G_F -$ open set in Y is $G_F - (i, j) -$ regular − open set in X
Definition 5.2: Let (X, T_1, T_2) and (Y, J_1, J_2) be G_F −bi-topological space. A mapping $F:(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is said to be somewhat pairwise G_F −completely continuous map if $A \neq 0_Y$ is a $J_i - G_F -$ open set on Y and $F^{-1}(A) \neq 0_X$ then there is a $G_F - (i, j) -$ regular − open set $B \neq 0_X$ such that $B \subseteq F^{-1}(A)$
Remark 5.1: Every G_F − ccontinuous map, G_F − semi-continuous map ($G_F - \alpha$-continuous map) and G_F − pre-continuous map ($G_F - \beta$-continuous map) is somewhat G_F −completely continuous map but converse may not be true which is illustrated in Example 5.1.

Example 5.1: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$.Consider fuzzy sets $A = \{(x_1/0.3), (x_2/0.6)\}$, $B = \{(x_1/0.5), (x_2/0.4)\}$, $C = \{(x_1/0.5), (x_2/0.6)\}$, $D = \{(x_1/0.2), (x_2/0.5)\}$, $E = \{(x_1/0.6), (x_2/0.4)\}$, $F = \{(x_1/0.5), (x_2/0.4)\}$ and $G = \{(x_1/0.6), (x_2/0.5)\}$ on X. Again $P = \{(y_1/0.3), (y_2/0.6)\}$, $Q = \{(y_1/0.5), (y_2/0.5)\}$, $R = \{(y_1/0.6), (y_2/0.6)\}$, $S = \{(y_1/0.2), (y_2/0.5)\}$, $T = \{(y_1/0.6), (y_2/0.3)\}$ and $U = \{(y_1/0.6), (y_2/0.5)\}$ on Y. Let $T_1 = \{0, A, B, C, 1\}$, $T_2 = \{0, D, E, F, G, 1\}$, $J_1 = \{0, P, Q, R, 1\}$ and $J_2 = \{0, S, T, U, 1\}$ be the G_F − topologies defined on X and Y. which is somewhat G_F − compltely continuous map. Now we define a mapping $F(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ such that $F(x_1) = y_1$ and $F(x_2) = y_2$. Now we can see that for $F^{-1}(Q) \neq 0_X$ and $G_F - (I, j) -$ regular − open set $B \neq 0_X$ such that $B \subseteq F^{-1}(Q) \neq 0_X$. Hence $F(X, T_1, T_2) \rightarrow (Y, J_1, J_2)$ is somewhat G_F − compltely continuous map but not G_F −ccontinuous map, G_F −semi-continuous map ($G_F - \alpha$-continuous map) and G_F −pre-continuous map ($G_F - \beta$-continuous map)

Proposition 5.1: Every somewhat fuzzy completely continuous map is somewhat fuzzy continuous map
Proof: Let X and Y be fuzzy topological spaces and $F: X \rightarrow Y$ be a somewhat fuzzy completely continuous map. Suppose $A \neq 0_Y$ is a fuzzy open set in Y and $F^{-1}(A) \neq 0_X$. Since "F" is a fuzzy completely continuous map, there is a fuzzy regular open set $B \neq 0_X$ in X such that $B \subseteq F^{-1}(A)$. As each fuzzy regular open set is fuzzy open, it follows that f is a somewhat fuzzy continuous map.
Remark 5.2: The converse of the Proposition 5.1 is not necessarily true as shown in Example 5.2
Example 5.2: Let A, B and C be fuzzy sets on $X = \{x_1, x_2, x_3\}$ and D and E be fuzzy sets on $Y = \{y_1, y_2, y_3\}$ which are defined as $A = \{(x_1/0.4), (x_2/0.4), (x_3/0.4)\}$ $B = \{(x_1/0.3), (x_2/0.3), (x_3/0.3)\}$, $C = \{(x_1/0.2), (x_2/0.2), (x_3/0.2)\}$, $D = \{(y_1/0.4), (y_2/0.3), (y_3/0.3)\}$ and $E = \{(y_1/0.3), (y_2/0.3), (y_3/0.2)\}$. Consider fuzzy topologies $T = \{0, A, B, C, 1\}$ and $J = \{0, D, E, 1\}$. Now we define a map $F:(X, T) \rightarrow (Y, J)$ such that $F(x_1) = F(x_2) = y_2$, $F(x_3) = y_3$.Thus we can see that for $F^{-1}(D) \neq 0_X$ and $F^{-1}(E) \neq 0_X$ there exist fuzzy open set $C \neq 0_X$ such that $C \subseteq F^{-1}(D) \neq 0_X$ and $C \subseteq F^{-1}(E) \neq 0_X$. Hence $F:(X, T) \rightarrow (Y, J)$ is somewhat fuzzy continuous map but not somewhat fuzzy completely continuous map.

6. Conclusion
In this Paper we have studied a new concept of somewhat pair-wise fuzzy completely continuous map in fuzzy bi-topological spaces in which many important results have been obtained with help of some examples. Further we introduce the new concept of somewhat pair − wise* fuzzy completely continuous map and have established the relationships with the help of some counter examples.
The results discussed in this paper are concluded as follows:

Fuzzy continuous map ⇒⇍ somewhat fuzzy continuous map
Somewhat fuzzy completely continuous map ⇒⇍ somewhat fuzzy continuous map
Pair-wise fuzzy continuous map ⇒⇍ pair-wise* fuzzy continuous map
Pair-wise* fuzzy completely continuous map ⇒⇍ pair-wise* fuzzy continuous map
Pair-wise* fuzzy completely continuous map ⇒⇍ somewhat pair-wise fuzzy completely continuous map

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