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Preliminary On Results Lattices

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1.Abstract : In the Paper Mainly we have obtained certain Preliminary results on Lattices and also we obtain certain characterization.

2.Introduction: Lattices Plays important role in all Branches of sciences and Engineering, A Lattice is a non-empty set defined on 'L' which satisfy.

3.Key Words: P.O. Set, Lattice cumulative ,Associative Independent sup & Inf of Lattices and absorption Laws under meet $\wedge$ and join $\vee$ in the result.

1. we have the Lattice under binary relation ' $\leq$ ' and also it is observed in theorem that when a Lattice is given under ordering ' $\leq$ ' under join ' $\vee$' and meet $\wedge$. If we Define a Lattice ' < ' under partial ordering ' $\leq$ ' then (<, $\leq$) is a unique Lattice which is obtained in theorems 3,4&5.

Def: Set (<,$\leq$)be a P.o.set with sup {a,b} and inf { a,b} exists for every a,b $\in$L. then the P.O. set in called a Lattice. Denoted (<,$\leq$) as Sup {a,b} = avb, inf { a,b} = a$\wedge$b.

Theorem Lattice: A P.o. set (<,$\leq$) is a Lattice iff there exists a finite subset H of ' L' such that VH and $\wedge$H exists

Proof: set (<,$\leq$) be a P.O. set in which any two elements have sup & inf . then (<,$\leq$) is a Lattice. Conversely let (<,$\leq$) be a Lattice and H is a subset of L.
A subset of L

Case: If H= {a} then VH=$\wedge$H.
If H= {a,b,c} we show that VH& $\wedge$H exists
Let d= Sup {a,b} and e= sup {d,c}
Then d$\geq$,d$\geq$b and e>d, e$\geq$,c.
e $\geq$a,b,c and Hence 'e' is the upper bound of {a,b,c}
Let 'f' be any upper bound of {a,b,c} then
f$\geq$a, f$\geq$a,f$\geq$b,f$\geq$c
$\Rightarrow$f$\geq$sup {a,b} and f$\geq$d, f$\geq$c.
$\Rightarrow$f$\geq$su,p {d,c} and hence f$\geq$e.
so that sup {a,b,c} = (avb)vc.
Similarly Inf {a,b,c} exists.
Denote sup {a,b}= a$\vee$b and Inf {a,b}= a$\wedge$b.
Where v and $\wedge$ are two binary Operations.
on L satisfy the following Laws.

1. **Commutative** : For a,b $\in$ <.
a$\vee$b + sup {a,b} = sup {b,a}= b$\vee$a.
2. **Associative:** For a,b,c$\in$<, (avb) vc= av (bvc)
Now (avb) vc = Sup{avb,c}
=Sup {a, b$\vee$c}
= Sup {a,b,c} (Claim)
a$\vee$ (b$\vee$C) = Sup {a, b$\vee$c}
= Sup {a, b, c} (claim)
Let d= Sup {a,b} , e= sup {d,c}
then d $\geq$a, d$\geq$b, e$\geq$d,e$\geq$c.
e$\geq$a,b,c 'e' is an U.B. of {a,b,c}.
Let 'f' be any U.B of {a,b,c}
f$\geq$a,b,c $\Rightarrow$ f$\geq$e.
Hence 'e' is Sup {a,b,c}
Similiarly a$\vee$ (bVC)= Sup {a,b,c}
Now we Claim that

$a, \vee a_2 \vee \ldots \vee$ an-1,$\vee$ an = Sup $\{a_1$, Sup{a2,a3….an-1and $a_n\}$
It n=1 then $a_{1=}$ Sup $\{a_1\}$ Hence it in Clear.
By using Mathematical introduction. Let it be true for n-1.
Sup {a1, a2…. $an_{-1,}$ an} = ($a_1 \vee a_2 \vee$---$\vee$ an-1) $\vee$ an
= Sup $\{a_1$, Sup $\{a_2$ –an-$_1\}$.
Hence is true for any finite elements
$\{a_{1,}a_2a\ldots a_n\}$ of L and hence
$a_1 \vee$a2 $\vee \ldots \vee$ an is uniquely determined.

Theorem 2: Let (L,≤) be a Lattice, where '≤' is a binary operation an ≤ satisfying Laws and transitive Laws and Sup {a,b) , Inf {a,b} exists for all a,b, in L:
Define two "binary operations $\vee$ and $\wedge$ in < in (<,$\vee$,$\wedge$)
by a$\vee$b = Sup { a,b} and . a$\wedge$b= inf {a,b} then (<,$\wedge$,$\wedge$) is a Lattice.

Theorem 3: Let < L,$\vee$,$\wedge$> be a Lattice, where $\vee$ and $\wedge$ are binary Operations on L, with the a$\vee$b= sup {a,b} and a$\wedge$b= Inf {a,b}.
Define a relation '≤' on ≤ by ≤ b' if a$\wedge$b=a then ≤L,≤≥ is a Lattice.

Proof : 1. Relative : a ≤ since a$\wedge$a= Inf {a,a}
2. Antisysmetic: Let a≤b and b≤a for any a,b$\in$L then
and for b≤a and a$\wedge$b= Inf {a,b}=$\in$a
Inf {b,a}= b$\wedge$a=b
Hence a=b '$\wedge$' is commutative and Inf {a,b}=Inf {b,a}.
3. Transitive: Let a≤b and b≤c.
Then we have to show that a≤c.
a≤b, Inf {a,b}-a
b≤c and Inf {b,c}= b$\wedge$c.
a= a$\wedge$b=a$\wedge$(b$\wedge$c)= (a$\wedge$b)$\wedge$c = a$\wedge$c.
Hence a≤c.Hence ≤ is Transitive.
Now we Claim that '≤' is a partial orders on <.
Now we show that Sup {a,b}= a$\vee$b, Inf {a,b}=a$\wedge$b.
Since a$\wedge$(a$\vee$b)=a⇒a≤a$\vee$b
b$\wedge$(a$\vee$b)=b⇒b$\vee$b.
hence a$\vee$b is an O.B. of {a,b}
Let 'd' be an O.B.of {a,b}
d≥a, d≥b⇒d≥a$\vee$b
hence a$\vee$b=sup{a,b}.
similarly a$\vee$ (a$\wedge$b)=a⇒a$\wedge$b≤a and b$\wedge$(a$\wedge$b)⇒a$\wedge$b≤b.
hence a$\wedge$b is Lower bound of {a,b}.
Let 'e' be any Lower bound of {a,b} then e≤a, e≤b⇒e≤a$\wedge$b.
Hence a$\wedge$b= Inf [a$\wedge$b=Inf {a,b} Imply that (L,≤) is Lattice.

Theorem 4: Let (L,$\vee$,$\wedge$) be a Lattice, where $\vee$ and $\wedge$ are two binary operations on L with a$\vee$b= sup {a,b} and Inf {a,b}=a$\wedge$b.
Define '≤' on ≤ by a≤b if a$\wedge$b=a. Then ≤≤,≤≥ is a Lattice and the Ordering '≤' is unique.

Proof: If '≤' be any binary operation defined on <,defined by a≤,b if a$\vee$b=b
Claim: $\leq_1$=≤
let (a,b) $\in$≤ iff a≤b if a$\vee$b=b
if a≤b if (a,b) □≤.
Imply that ≤,≤,≤→⊗
Let (a,b) $\in$≤, iff a$\leq_1$b iff Inf {a,b}=a
Iff a≤b iff (a,b)$\in$≤ imply that
≤=$\leq_1$ $\leq_{1,}$ ≤ ≤→⊗ ⊗
For an ⊕and ⊗ ⊗≤=≤.it is also observed that a∇b= a$\vee$b and a$\wedge$b=a$\wedge$b.
Now we claim that ≤ is a partial on dew on ≤
Now we s.t. sup {a,b} =ayb

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Inf {a,b} = a∧b.
Since a∧(a∨b) =a⇒a≤a∨b,
b∧(a∨b)=b⇒b≤a∨b
hence a∨b is an O.B. of {a,b}
Let 'd' be an U.B.of {a,b}
d≥a,d≥b⇒d≥a∨b
hence a∨b=sup {a,b}, a∧b≤a.
similarly a ∨(a∧b)=a⇒
and b∨(a^b)=b⇒a∧b≤b.
hence a∧b is Lower bound of [a,b}
let 'e'be any lower bound of {a,b}
then e≤a, e ≤b⇒ e≤a∧b.
hence a∧b=Inf {a,b} imply that (≤,≤) is a Lattice.

Theorem 5: Let (<,∨,∧) be a Lattice, where ∨ and ∧ are two binary operations on L with a∨b= sup {a,b} and a∧b = Inf {a,b}.
Define: < on ≤ by a ≤b if a∧b =a then (<,≤) is a Lattice
And the ordering ≤ is Unique.
Proof: If '$\leq_1$' be any binary operation on < defined by a$\leq_1$b iff a∨b=b
Claim: $\leq_1 = \leq_1$
Let (a,b) ∈≤ if a≤b if a∨b=b
If a≤b if (a,b) if (a,b)□≤
Imply that ≤ ≤ ≤ →
Let (a,b) ∈≤, if a≤b if
Inf {a,b}=a
Inf a≤b if (a,b)∈≤
Imply that ≤,≤ ≤ → ⊗ ⊗
F or an ⊕ and ⊗ ⊗ ≤ =$\leq_1$
It in also Observed that
a∇v=a∨b and a∧b=a∧b
The following is an Example of a p.o. set in which supreme of the set consisting of any two elements exists and infimum doesn't exists.

Example 1: Let x = Infinite set. P= Set of all non-employ is subsets of x Define ≤ as 'c' (i.e) for A,B□P, A≤B IF A≤B iff A≤B
For A,B □p ⇒A≠Q, B≠Q ⇒ AUB≠Ø
So that AUB∈P
Now we claim that A∨B=AUB
Since A ≤ AUB and B ≤ AUB ⇒AUB is an U.B OF {A,B}
Let 'd'be any U.B.of {A,B} Then D≥A,D≥B so that AUB=AVB
Let A,B be any two non-empty subsets of x such that A∩B=Ø
Let'□' p be any lower bound of {A,B}
Then P≤A, P≤B⇒P≤A∩B⇒P=Ø
Hence A∩B doesn't exist.
The following is an example of a p.o. set in which Infinimum of the set consisting of any two elements exists and supreme doesn't exists.

Example 2: Set x= infinite set and P=the set of all subsets of X.
Let ≤ be a partial ordering on 'P' now we S.T. A∧B and AUB doesn't exist for any A,B□P.
Since A∩B≤A, A∩B≤B⇒ A∩B is a lower bound of {A,B}
Let A,B beany two subsets of 'X' such that
AUB=x
Let c∈p be any U.B. OF {A,B}
⇒AUB≤C.
Let A≤c,B≤C⇒ AUB≤C so that X ≤ C
Hence X=C. which is a contradiction as C∈P.
So that AUB doesn't exists in P.

Def 2: covering of two elements in a P.O. se let (p.≤) be a p.o. set and let a,b∈p than we say that 'a'covers 'b' b is covered by a if 1. b≤a 2. There exists x∈p.
such that b≤x≤a we write this as b≤a.
Theorem 6 : Let (P,≤) be a finite P.O. set and set a,b□p then a≤b if a=b
There exists a finite sequence {x_0=a, x_{n-1}=b and x_i ________ ≤ x_{i+1} for 0≤i≤n-1
Proof: Let (p,≤) be a finite p.o. set and let a,b∈p. If the exists a finite sequence a=x_0 ≤ x_1 ≤ - - -≤ x_{n-1}=b, then a≤b.
Let (p,≤) be a P.O. set with a≤b∇a,b∈p.
If a=b then it is clear.
If a≤b: then we have to construct a since sequence {a=$x_{0,}$$x_1$ ----x_{n-1}=b}
Let 'H' be any subset of 'P' which in contains non-elements say {$x_{0,}$$x_1$-----$x_{m-1\}}$
With x_0 =a, as least element and x_{n-a1} =b as greatest element a=x_0 ≤ x_1 ≤----≤ x_{n-1}
Of $x_i \neq x_i$ +1, x∈p ,x_i≤x≤x_i+1
Then HU {x} contains+1 elements which is a contradiction as H contains m elements and hence x_i--------------≤x_i+1.

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