

Construction And Selection Of A Bayesian Single Sampling Scheme With A Gamma Prior Distribution Using Weighted Poisson Distribution

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Abstract

The purpose of this paper present a method for creating and choosing a Bayesian single sampling scheme with a gamma prior distribution using weighted Poisson distribution. The weighted Poisson function is used to derive the average probability of lot acceptance based on defective random defective products. Gamma distribution is considered as a suitable prior for weighted Poisson distribution. It is discussed with i) Acceptance Quality Level and Limiting Quality Level, ii) control point and the relative slope at that point and iii) MAPD. Tables are provided for the easy selection of the plans.

Keywords and Phrases – Bayesian sampling scheme, Single sampling scheme, Acceptance quality level, Limiting quality level, Maximum allowable percentage defective, Operating Characteristic.

AMS (2000) Subject Classification Number: Primary: 62P30 Secondary: 62D05

1. Introduction

The use of prior process history for the selection of Gamma Poisson distributions to characterize the random fluctuations involved in Acceptance sampling is known as the Bayesian Acceptance Sampling technique. With Bayesian sampling strategies, the user must explicitly state the lot distribution. The predicted distribution of batch quality on which the sampling strategy operates is the errors of the batch to the prior distribution. This distribution is known as previous since it was created before samples were collected. The decisions of the lot are determined by the interaction of previous information, illustrated by prior distribution, and empirical knowledge based on the sample.

Evaluation of a sample of a product to determine whether to accept or reject the full batch is known as acceptance sampling. Since Hamaker (1960), a significant amount of work has been done with relation to the selection of sample plans for conventional sampling plans. Hald (1967) investigated how to determine a single sample characteristics strategy based on the risks of the consumer. To reach the minimal sample size required by the Bayesian sampling plan, Oliver and Springer (1972) proposed a set of tables based on the presumption of a Beta earlier dispersion with a specific back chance, which gets around the issue of deciding fetched parameters. Procedures and tables for carrying out Bayesian sampling schemes are provided by Calvin (1984). For situations where the fraction of defects has a Beta prior distribution, Lawer (1979) provided a method for determining acceptance probability. Wise (1946) described the distribution of the number of pinholes in the enameled wire using the Gamma-Poisson distribution. Bayesian sampling strategies using a Gamma prior have been investigated by Suresh and Latha (2002) and Pradeepa Veerakumari (2007). Using a weighted Poisson distribution Mohana Priya (2008) created a Bayesian single sampling plan with gamma prior. Yuvaraj (2016) demonstrates that the Bayesian optimum single sampling plan for an attribute with a prior binomial distribution. Latha and Palanisamy building a Bayesian Single Sampling Plan by Attributes in the Presence of an Inflated Poisson Distribution under Gamma Zero Conditions. The major topic of the presentation by Suresh, Umamaheswari, and K. Pradeepa Veerakumari (2013) is how to create a Bayesian Repetitive Deferred Sampling Plan that is indexed with relative slopes at the Acceptable, Limiting, and Indifference Quality Levels. Designing a Bayesian Two-Sided Group Chain Sampling Plan for Gamma Prior Distribution was done by Waqar Hafeez and Nazrin Aziz in 2021

In this section, instructions and tables are provided for choosing a Bayesian single sampling scheme that uses a weighted Poisson distribution and any of the input parameter combinations listed below:

i) AQL and LQL

ii) Point of control and relative slope at that point.

iii) MAPD indicated as μ_* and $K = \mu_T / \mu_*$ where μ_T is the esteem of μ , the normal percent inadequate at which the digression to the normal likelihood of acknowledgment (APA) bend at the point of emphasis cuts the μ axis.

2. Glossary of symbols

N : lot size

n : sample size for each lot

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Pa (p) : probability of acceptance for given quality

c : acceptance number d : number of defectives in the sample

 h_0 : relatives slope at *po*

 h_{*} : relative slope at p^{*}

 μ : delivered quality of a batch or process.

μ^{*} : maximum allowable percent defective

 μ_1 : the submitted quality level such that Pa (p1) = 0.95

 μ_2 : the submitted quality level such that Pa (p2) = 0.10

 μ_0 : the submitted quality level such that Pa (p0) = 0.50

 μ_T : value μ at which the line tangent to the average acceptance probability curve OC at the inflection point intersects the μ axis.

AQL : acceptable quality level

LQL : limiting quality level

MAPD : maximum allowable percent defective

3. Process of Single sampling scheme

The working process of the single sampling scheme is as follows.

> Take a random sample from a lot of size n and count the number of defects in the sample.

- Accept the lot, if $d \le c$
- \blacktriangleright Reject the lot if d > c.

Flow chart



4. Average probability of acceptance curve (APA curve)

The OC function is provided as follows for a single sampling strategy using a weighted Poisson model,

P (p/n, c) =
$$\sum_{x=1}^{c} \frac{e^{-np} (np)^{x-1}}{(x-1)!}$$

Assume that p has a density function depending on the prior history of inspection and a gamma prior distribution,

W (p) =
$$\frac{\beta^{\alpha} p^{\alpha-1} e^{-\beta p}}{\Gamma \alpha}, \alpha, t, p \ge 0$$

where $\alpha \& \beta$ are the parameters with mean $\mu = \frac{\alpha}{\beta}$. The average probability of acceptance for the gamma prior distribution, according to Hald (1981), is

$$\overline{p}(\mu) = \int_{0}^{\infty} P(p/n,c) w(p) dp$$

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5. Construction of Tables

Equation (1) provides the necessary likelihood of acceptance, and a Java program is used to determine the nµ values (in Table 1).

The OC curve h₀'s relative slope is determined by, $h_0 = \left(\frac{-\mu}{\overline{p(\mu)}}\right) \frac{d\overline{p(\mu)}}{d(\mu)}$ at $\mu = \mu_0$ (in Table 2) by using Java

program.

The values are presented in Table 3 are obtained $p(\mu)$ in equation (1) is differentiated twice and the second order derivative is equated to zero which gives $n\mu = \alpha(c-1)/(\alpha+1)$.

The OC curve h*'s relative slope is determined by, $h_* = \left(\frac{-\mu}{\overline{p}(\mu)}\right) \frac{d\overline{p}(\mu)}{d(\mu)}$ at $\mu = \mu_*$.

6. Selection of Bayesian Single sampling scheme for given AQL, LQL's

Table 2 is utilized to construct the Bayesian single sampling scheme when $AQL = \mu_1$, $LQL = \mu_2$ and α , the parameters of prior distribution are specified. From the given values of μ_1 and μ_2 the operating ratio = μ_2 / μ_1 is calculated c and $n\mu_1$ corresponding to μ_2 / μ_1 and α are found out from Table 2. One can find the value of n by dividing $n\mu_1$ by μ_1 .

Example 1

Assume that the gamma prior distribution's parameter α is 7, $\mu_1 = 0.0156$, and $\mu_2 = 0.1300$. This results in an operational ratio of $(\mu_2 / \mu_1) = 8.3333$. From Table 2, the values of c, corresponding to $(\mu_2 / \mu_1) = 8.5742$ which is nearer to 8.3333 and $\alpha = 7$ is c = 4. When $\alpha = 7$ and c = 4 the value of $n\mu_1 = 0.7880$ which gives $n = (n\mu_1 / \mu_1) = 51$. Hence the required plan has parameters n=51, c = 4. The operating characteristic curve for the plan is displayed in Figure 1.



Example 2

Assume the parameter α of the gamma prior distribution is 5, $\mu_1 = 0.005$ and $\mu_2 = 0.035$ which gives operating ratio (μ_{2/μ_1}) = 7.0000. From table 2, the values of c, corresponding to (μ_{2/μ_1}) = 7.7883 which is nearer to 7.0000 and $\alpha = 5$ is c = 5. When $\alpha = 5$ and c = 5 the value of $n\mu_1 = 1.2245$ which gives $n = (n\mu_1/\mu_1) = 245$. Hence the required plan has parameters n = 245, c = 5.

7. Selection of plan for specified values of relative slope at that moment and the control point

Table 2 is applied to construct such a plan with μ_0 , the point of control h_0 the relative slope and parameter α of the prior distribution is specified. The value of $n\mu_0$ corresponding to α and h_0 is obtained from Table 2 and dividing it by μ_0 , the sample size, n, is determined. The value of c, the acceptance number is read directly from the Table 2.

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Example 3

It is given that $\alpha = 3$, $\mu_0 = 0.040$ and $h_0 = 1.3777$. From Table 2 the required plan is obtained with parameters n = 104, c = 5.

8. Selection of the plan when μ * and K are specified

Table 3 is used to construct the plan when MAPD μ_* , K, α are specified. The given values of α and K the corresponding $n\mu_*$ value is read from Table 3 from which n sample size can be calculated and c is obtained directly from Table 3.

Example 4

When $\alpha = 4$, K = 2.3068 and $\mu_* = 0.035$ using Table 3 values of n and c are obtained as 69, 4 respectively.

Practical Example

In the bolt manufacturing company, if the producer fixes the quality level as $\mu_1 = 0.0156$ (156 defectives out of 1000) and $\mu_2 = 0.1300$ (13 defectives out of 1000) and $\alpha = 7$.



Select a random sample of size 51 from the assembly line and count the number of non-conforming bolts (d). If $d \le 4$, accept the lot; otherwise, reject the lot. These plans definitely help the producers because of the smaller sample size, which directly results a lower sampling cost and indirectly reduces the total cost of the product.

9. Conclusion

The average probability of the acceptance function for a Bayesian single sampling design with prior Gamma was also obtained in this study using the weighted formulation of the inflection point of the Poisson distribution and the tangent as the inflection point. There is also an explanation of the selection procedure for the Bayesian single-sample attribute design (with gamma prior using a weighted Poisson distribution) based on the AQL and LQL, the control point, and the slope from this point, MAPD and K measure for sharpness. In addition, tables for quick selection of models are provided. These tables will aid the floor engineers in making judgments quickly based on the incoming and exiting quality factors. Typically, Bayesian designs have the same risks for producers and consumers requiring smaller sample sizes than conventional sampling plans. These systems do help producers, as the reduced sample size leads to cheaper sampling costs both directly and indirectly in the overall price of the product.

Table 1 nµ values for given average probability of acceptance by Bayesian SSP

α	c	0.99	0.95	0.75	0.50	0.10	0.05
1	2	0.0401	0.0826	0.3633	1.0299	9.0251	19.0101
1	3	0.1409	0.3179	1.0299	2.4439	18.5069	38.4837
1	4	0.3041	0.6129	1.7322	3.8758	27.9916	57.9587
1	5	0.4917	0.9267	2.4439	5.3144	37.4771	77.7341
1	6	0.6904	1.2482	3.1593	6.7540	46.9629	96.9096
1	7	0.8949	1.5737	3.8768	8.1946	56.4488	116.3852
1	8	1.1029	1.9015	4.5952	9.6370	65.9348	135.8609
1	9	1.3131	2.2308	5.3144	11.0772	75.4209	155.3365
2	2	0.0401	0.0820	0.3394	0.8583	4.3530	6.9699
2	3	0.1549	0.3429	0.9988	2.0298	8.2418	12.7987
2	4	0.3573	0.6914	1.7084	3.2147	12.0554	18.5085
2	5	0.5999	1.0719	2.4333	4.4028	15.8448	24.1800
2	6	0.8628	1.4671	3.1647	5.5923	19.6232	29.8345
2	7	1.1372	1.8703	3.8997	6.7823	23.3959	35.4798
2	8	1.4184	2.2784	4.6367	7.9730	27.1649	41.1197
2	9	1.7042	2.6897	5.3751	9.1636	30.9319	46.7560
3	2	0.0401	0.0817	0.3319	0.8097	3.4923	5.1706
3	3	0.1642	0.3544	0.9929	1.9136	6.3898	9.0931
3	4	0.3837	0.7299	1.7132	3.0297	9.1913	12.8757



3	5	0.6569	1.1467	2.4528	4.1486	11.9592	16.6098
3	6	0.9571	1.5836	3.2013	5.2687	14.7111	20.3208
3	7	1.2731	1.8703	3.9547	6.3893	17.4541	24.0191
3	8	1.5990	2.4874	4.7112	7.5101	20.1917	27.7095
3	9	1.9318	2.9475	5.4696	8.6314	22.9257	31.3947
4	2	0.0404	0.0816	0.3283	0.7868	3.1423	4.4869
4	3	0.1648	0.3609	0.9912	1.8591	5.6415	7.7025
4	4	0.3996	0.7532	1.7190	2.9431	8.0332	10.7675
4	5	0.6925	1.1930	2.4694	4.0297	10.3852	13.7772
4	6	1.0173	1.6573	3.2306	5.1173	12.7175	16.7595
4	7	1.3619	2.1357	3.9979	6.2055	15.0387	19.7264
4	8	1.7189	2.6231	4.7690	7.2938	17.3527	22.6833
4	9	2.0845	3.1167	5.5427	8.3825	19.6621	25.6338
5	2	0.0401	0.0816	0.3261	0.7734	2.9537	4.1311
5	3	0.1672	0.3652	0.9905	1.8275	5.2395	6.9835
5	4	0.4103	0.7686	1.7239	2.8928	7.4103	9.6781
5	5	0.7168	1.2245	2.4823	3.9608	9.5368	12.3124
5	6	1.0596	1.7082	3.2529	5.0296	11.6408	14.9162
5	7	1.4248	2.2085	4.0307	6.0989	13.7318	17.5022
5	8	1.8049	2.7193	4.8129	7.1687	15.8143	20.0766
5	9	2.1952	3.2375	5.5983	8.2385	17.8910	22.6433
6	2	0.0401	0.0815	0.3247	0.7647	2.8361	3.9137
6	3	0.1688	0.3682	0.9903	1.8067	4.9892	6.5460
6	4	0.4180	0.7796	1.7278	2.8601	7.0217	9.0154
6	5	0.7348	1.2476	2.4922	3.9158	9.0067	11.4206
6	6	1.0911	1.7459	3.2702	4.9725	10.9669	13.7927
6	7	1.4722	2.2626	4.0561	6.0296	12.9124	16.1441
6	8	1.8699	2.7914	4.8471	7.0870	14.8484	18.4844
6	9	2.2795	3.3286	5.6416	8.1446	16.7778	20.8150
7	2	0.0401	0.0815	0.3237	0.7586	2.7558	3.7674
7	3	0.1700	0.3704	0.9901	1.7923	4.8184	6.2524
7	4	0.4238	0.7880	1.7310	2.8370	6.7565	8.5705
7	5	0.7485	1.2652	2.5001	3.8841	8.6441	10.8215
7	6	1.1153	1.7748	3.2838	4.9322	10.5053	13.0370
7	7	1.5091	2.3046	4.0762	5.9807	12.3503	15.2312
7	8	1.9211	2.8746	4.8743	7.0296	14.1848	17.4113
7	9	2.3460	3.4000	5.6763	8.0785	16.0120	19.5815

Table 2 Certain Parametric values of Bayesian SSP

α	c	nμ ₁	nμ₀	nµ2	μ ₂ / μ ₁	h ₀
1	2	0.0826	1.0299	9.0251	109.2627	1.0074
1	3	0.3179	2.4439	18.5069	58.2161	1.2096
1	4	0.6129	3.8758	27.9916	45.6707	1.1882
1	5	0.9267	5.3144	37.4771	40.4415	1.2156
1	6	1.2482	6.7540	46.9629	37.6245	1.2942
1	7	1.5737	8.1946	56.4488	35.8701	1.3246
1	8	1.9015	9.6370	65.9348	34.6752	1.3472
1	9	2.2308	11.0772	75.4209	33.8100	1.3647
2	2	0.0820	0.8583	4.3530	53.0854	0.7146
2	3	0.3429	2.0298	8.2418	24.0356	0.8390
2	4	0.6914	3.2147	12.0554	17.4362	1.1133
2	5	1.0719	4.4028	15.8448	14.7820	1.4045
2	6	1.4671	5.5923	19.6232	13.3755	1.7001
2	7	1.8703	6.7823	23.3959	12.5092	1.9971
2	8	2.2784	7.9730	27.1649	11.9228	2.2947
2	9	2.6897	9.1636	30.9319	11.5001	2.5926
3	2	0.0817	0.8097	3.4923	42.7454	0.6142



3	3	0.3544	1.9136	6.3898	18.0299	0.6952
3	4	0.7299	3.0297	9.1913	12.5925	0.9838
3	5	1.1467	4.1486	11.9592	10.4292	1.3777
3	6	1.5836	5.2687	14.7111	9.2897	1.8679
3	7	1.8703	6.3893	17.4541	9.3322	2.4518
3	8	2.4874	7.5101	20.1917	8.1176	3.1238
3	9	2.9475	8.6314	22.9257	7.7780	3.8999
4	2	0.0816	0.7868	3.1423	38.5086	0.5704
4	3	0.3609	1.8591	5.6415	15.6318	0.6200
4	4	0.7532	2.9431	8.0332	10.6654	0.8854
4	5	1.1930	4.0297	10.3852	8.7051	1.3117
4	6	1.6573	5.1173	12.7175	7.6736	1.9221
4	7	2.1357	6.2055	15.0387	7.0416	2.7454
4	8	2.6231	7.2938	17.3527	6.6153	3.8117
4	9	3.1167	8.3825	19.6621	6.3086	5.1512
5	2	0.0816	0.7734	2.9537	36.1973	0.5476
5	3	0.3652	1.8275	5.2395	14.3469	0.5785
5	4	0.7686	2.8928	7.4103	9.6413	0.8143
5	5	1.2245	3.9608	9.5368	7.7883	1.2389
5	6	1.7082	5.0296	11.6408	6.8147	1.9135
5	7	2.2085	6.0989	13.7318	6.2177	2.9145
5	8	2.7193	7.1687	15.8143	5.8156	4.3289
5	9	3.2375	8.2385	17.8910	5.5262	6.2537
6	2	0.0815	0.7647	2.8361	34.7988	0.5343
6	3	0.3682	1.8067	4.9892	13.5502	0.5541
6	4	0.7796	2.8601	7.0217	9.0068	0.7620
6	5	1.2476	3.9158	9.0067	7.2192	1.1689
6	6	1.7459	4.9725	10.9669	6.2815	1.8695
6	7	2.2626	6.0296	12.9124	5.7069	2.8562
6	8	2.7914	7.0870	14.8484	5.3193	4.6904
6	9	3.3286	8.1446	16.7778	5.0405	7.1628
7	2	0.0815	0.7586	2.7558	33.8135	0.5258
7	3	0.3704	1.7923	4.8184	13.0086	0.5387
7	4	0.7880	2.8370	6.7565	8.5742	0.7723
7	5	1.2652	3.8841	8.6441	6.8322	1.1047
7	6	1.7748	4.9322	10.5053	5.9191	1.8061
7	7	2.3046	5.9807	12.3503	5.3590	2.9993
7	8	2.8746	7.0296	14.1848	4.9345	4.9188
7	9	3.4000	8.0785	16.0120	4.7094	7.8713

Table 3 : MAPD, K, h* for Bayesian SSP

α	с	nµ∗	h*	K
1	2	0.5000	0.8330	2.200
1	3	1.0000	0.8925	2.1204
1	4	1.5000	0.9531	2.0492
1	5	2.0000	1.0435	1.9583
1	6	2.5000	1.1099	1.9009
2	2	0.6667	0.6667	2.4999
2	3	1.3333	0.6905	2.4482
2	4	2.0000	0.8571	2.1667
2	5	2.6667	1.0324	1.9686
2	6	3.3333	1.2068	1.8286
3	2	0.7500	0.6042	2.6550
3	3	1.5000	0.6250	2.6000
3	4	2.2500	0.8046	2.2429
3	5	3.0000	1.0333	1.9678
3	6	3.7500	1.3034	1.7672



4	2	0.8000	0.5720	2.7483
4	3	1.6000	0.5871	2.7033
4	4	2.4000	0.7652	2.3068
4	5	3.2000	1.0246	1.9758
4	6	4.0000	1.3709	1.7294
5	2	0.8333	0.5529	2.8086
5	3	1.6667	0.5632	2.7755
5	4	2.5000	0.7336	2.3631
5	5	3.3333	1.0078	1.9923
5	6	4.1670	1.4101	1.7092
6	2	0.8571	0.5406	2.8498
6	3	1.7143	0.5473	2.8272
6	4	2.5714	0.7075	2.4134
6	5	3.4286	0.9858	2.0144
6	6	4.2857	1.4256	1.7015
7	2	0.8750	0.5321	2.8793
7	3	1.7500	0.5363	2.8646
7	4	2.2650	0.6348	2.5758
7	5	3.5000	0.9610	2.0406
7	6	4.3750	1.4238	1.7024

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